(Adding and Subtracting Fractions)

Ground Rules for Problem Set Completion

- 1. Present your work in a neat and organized manner. Use <u>complete sentences</u> whenever you are asked to make a statement.
- 2. SHOW YOUR WORK: Credit is awarded for all reasonable attempts, based on the work shown.
- 3. Make sure you answer ALL parts of problems.
- 4. Complete and submit ALL Problem Sets for the unit prior to taking the Unit Test.

I. REVIEW PROBLEMS

The problems below provide practice with skills and concepts covered in Problem Set A1.

A. Write a fraction that represents the shaded portion of each figure.



- B. Write fractions for each of the *parts* described below. Give your answer in lowest terms.
 - 1. Three nickels are what fraction of a dollar?
 - 2. What fraction of John and Amy's \$2550 monthly take-home pay goes for rent if they pay \$550 for rent each month?
 - 3. What fraction of a mile is 2000 feet? There are 5,280 feet in a mile.
- C. Tell whether each number is a **proper fraction**, **improper fraction**, or a **mixed number**. 1. $\frac{8}{5}$ 2. $\frac{5^2}{3}$ 3. $\frac{11}{13}$ 4. $\frac{8}{15}$ 5. $\frac{20}{7}$
- D. Change each **improper fraction** to a **whole number** or a **mixed number** in lowest terms. 1. $\frac{9}{4}$ 2. $\frac{12}{4}$ 3. $\frac{16}{12}$ 4. $\frac{85}{45}$ 5. $\frac{100}{25}$
- E. Change **each mixed number** to an **improper fraction** in lowest terms. 1. $1^{1}/_{3}$ 2. $5^{8}/_{10}$ 3. $6^{5}/_{20}$ 4. $10^{3}/_{12}$ 5. $2^{6}/_{11}$

F. Create equivalent fractions by raising each fraction to the specified higher term. 1. $\frac{2}{5} = \frac{2}{15}$ 2. $\frac{7}{8} = \frac{2}{32}$ 3. $\frac{9}{16} = \frac{2}{96}$ 4. $\frac{17}{20} = \frac{2}{100}$

G. Measure each line to the nearest $\frac{1}{16}$ inch. <u>Give your answer in lowest terms</u>.

1. _____

- 2.
- H. Add each set of fractions and <u>reduce to lowest terms</u>. 1. ${}^{3}/_{8} + {}^{1}/_{8} =$ 2. ${}^{4}/_{15} + {}^{2}/_{15} =$ 3. ${}^{1}/_{9} + {}^{2}/_{9} + {}^{3}/_{9} =$ 4. ${}^{3}/_{25} + {}^{6}/_{25} + {}^{11}/_{25} =$
- I. Subtract each pair of fractions and reduce to lowest terms.
 - 1. $\frac{5}{6} \frac{1}{6} =$ 2. $\frac{13}{18} \frac{7}{18} =$ 3. $\frac{31}{39} \frac{5}{39} =$ 4. $\frac{67}{100} \frac{23}{100} =$

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- J. Completely solve each problem using steps (i), (ii), and (iii) below. Reduce all fractions.
 - *i*. State what it is you are to find. Give your answer as a complete sentence.
 - *ii*. Solve the problem, showing your work.
 - *iii*. State the answer in a complete sentence.
 - 1. On the first day of her trip to visit her parents Julie covers $\frac{4}{20}$ of the total trip distance. The second day she only covers $\frac{2}{20}$ of the total trip distance because she stopped to visit a friend. To make up for lost time, she travels $\frac{6}{20}$ of the total trip distance on the third day. What fraction of the total trip distance has she completed by the end of the third day?
 - 2. Jeremy cuts $8^{3}/_{8}$ inches from a $36^{7}/_{8}$ long board. How long is the remaining piece of wood?

II. DIVISIBILITY RULES

Both when **reducing fraction** and when **finding common denominators** it is helpful to be able to tell at a glance whether one number can be divided evenly by another. There are many **divisibility rules**; however, for now we will concern ourselves with just three basic rules: **divisibility by 2**, **divisibility by 3**, and **divisibility by 5**. These three rules will allow us to check for divisibility by many common factors, since we can use them sequentially or in combination. (For example, a number that is divisible by 2, 3, and 5 is also divisible by $30 - 2 \times 3 \times 5 = 30$.)

The rules for determining if a number can be divided by 2, 3, or 5 are given below:

- A number is **divisible by 2** if it is an **even number**. (Example: 58 is even, therefore it is divisible by 2.)
- A number is **divisible by 3** if the **sum of its digits is divisible by 3**. (Example: The sum of the digits of 528 is 15 [5 + 2 + 8 = 15]. 15 is divisible by 3, therefore 528 is divisible by 3.)
- A number is **divisible by 5** if **it ends in a 0 or 5**. (Example: 25,670 ends in a 0, therefore it is divisible by 5.)
- A. Use the divisibility rules to state whether each of the numbers below is divisible by 2.
 - 1. 30 2. 42 3. 35
- B. Use the divisibility rules to state whether each of the numbers in Problem A above is divisible by 3.
- C. Use the divisibility rules to state whether each of the numbers in Problem A above is divisible by 5.

III. ADDING & SUBTRACTING FRACTIONS & MIXED NUMBERS WITH UNLIKE DENOMINATORS

As discussed in Sections V and VI of Problem Set A1, **fractions can only be added or subtracted if they have the same denominators**. There are several strategies that can be used to find **Least Common Denominators (LCD)**. For now, however, we will focus on just two.

- 1. Check to see if the largest denominator is divisible by each of the smaller denominators. If it is, then the largest denominator is the Least Common Denominator.
- 2. List several multiples of each denominator. The smallest number that appears in all the lists is the LCD.

Although these approaches are not always the fastest, they will enable you to find the LCD of any fractions.

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SAMPLE PROBLEM 1 DEMONSTRATES HOW TO USE THESE STRATEGIES TO FIND LCDS.

SAMPLE PROBLEM 1 WITH SOLUTION			
Find the Least Common Denominator (LCD) of each set of the fractions below.			
a. $3/4$, $7/16$, & $13/32$	 Solution: Check to see if the largest denominator is divisible each of the smaller denominators. ▶ 32 is divisible by both 16 (2 times) and 4 (8 times), therefore 32 is the LCD. 	by	
b. $2/5$, $5/8$, and $7/20$	 Solution: 1. Check to see if the largest denominator is divisible each of the smaller denominators. ▶ 20 is NOT divisible by 8, thus 20 is NOT the LC 	by D.	
	2. List several multiples of each denominator. (I usually start by listing 4 or 5 multiples of the largest denominator. I can always add to the list, if needed.)		
	Multiples of 20: 20, <u>40</u> , 60, 80		
	Multiples of 8: 8, 16, 24, 32, <u>40</u>		
	Multiples of 5: 5, 10, 15, 20, 25, 30, 35, <u>40</u>		
	 40 is the smallest number that appears in all thread lists, therefore 40 is the LCD. 	e	

A. Find the Least Common Denominator (LCD) of each set of the fractions. Refer to Sample Problem 1 and the examples on pages 25 & 26 of Contemporary's <u>Number Power 2</u>, as needed. 1. $\frac{1}{4} \& \frac{5}{8}$ 2. $\frac{2}{9} \& \frac{1}{6}$ 3. $\frac{3}{5} \& \frac{3}{8}$

SAMPLE PROBLEM 2 DEMONSTRATES HOW TO ADD AND SUBTRACT FRACTIONS AND MIXED NUMBERS WITH UNLIKE DENOMINATORS.



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B. Perform the indicated addition or subtraction then <u>reduce the resulting fractions or mixed</u> <u>numbers to lowest terms</u>. Refer to Sample Problem 2, as needed. For more practice, see pages 32 and 33 of Contemporary's <u>Number Power 2</u> work-text.

23 40	3 10	25 9
$\frac{-\frac{5}{16}}{1}$	2. $+ \frac{8}{15}$	$-19\frac{2}{5}$
4. $\frac{1}{2} + \frac{2}{5} =$	5. $7/_8 - 1/_4 =$	6. $10^5/_8 + 5^1/_3 =$

IV. CARRYING WHEN ADDING FRACTIONS & MIXED NUMBERS

Often when adding fractions and mixed numbers the result contains an improper fraction. In these cases we change the improper fraction to a mixed number then add the resulting whole number portion to the whole number portion of the original answer.

SAMPLE PROBLEM 3 DEMONSTRATES CARRYING WHEN ADDING FRACTIONS AND MIXED NUMBERS.

SAMPLE PROBLEM 3 WITH SOLUTION			
Perform the indicated addition then reduce the resulting fractions or mixed numbers to lowest terms. a. $\frac{5}{6}$ $+ \frac{7}{15}$	 Solution Steps: > Write the problem in vertical form, > find a common denominator and convert the numerators, > add the fraction portions and reduce to lowest terms, > convert the resulting improper fraction to a mixed number, > add the whole numbers (including from the step above), > combine the resulting whole number and fraction portions. 		
$\frac{\frac{5}{6}}{\frac{6}{6}} = \frac{\frac{5}{6}}{\frac{5}{5}} \times \frac{\frac{5}{5}}{\frac{5}{5}} = \frac{\frac{25}{30}}{\frac{30}{5}}$ $\frac{+\frac{7}{15}}{\frac{15}{15}} = \frac{+\frac{7}{15}}{\frac{15}{2}} \times \frac{2}{\frac{2}{2}} = \frac{+\frac{14}{30}}{\frac{39}{30}} =$	$\frac{2\frac{5}{8}}{10} = 1\frac{3}{10} = 1\frac{3}{10}$ $\frac{2\frac{5}{8}}{10} = \frac{2\frac{5}{8} \times \frac{2}{2}}{16} = \frac{2\frac{10}{16}}{\frac{7}{16} \times \frac{1}{1}}{\frac{1}{16} \times \frac{1}{1}} = \frac{2\frac{10}{16}}{\frac{17}{16} \times \frac{1}{16}} = \frac{1}{16} + 1\frac{1}{16} = 7\frac{1}{16}$		

A. Perform the indicated addition, then <u>reduce the resulting fractions or mixed numbers to lowest</u> <u>terms</u>. Refer to Sample Problem 3, as needed. For more practice, see pages 25 – 28 of Contemporary's <u>Number Power 2</u> work-text.

$$\frac{\frac{3}{4}}{\frac{5}{8}}$$

$$\frac{5}{2}\frac{2}{3}$$

$$\frac{5}{2}\frac{2}{3}$$

$$\frac{13}{\frac{3}{8}}$$

$$\frac{5}{2}\frac{2}{3}$$

$$\frac{13}{\frac{3}{8}}$$

$$\frac{13}{\frac{3}{8}}$$

$$\frac{13}{\frac{9}{20}}$$

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V. BORROWING WHEN SUBTRACTING MIXED NUMBERS

Sometimes when subtracting fractions and mixed numbers we have a situation where the fraction we are subtracting is larger than the one we are subtracting from. In these cases we need to borrow a unit from the mixed number we are subtracting from. We need to remember to borrow based on the common denominator, then add what we borrow to the original numerator. For example, if the LCD is 12 we borrow $1^{12}/12$ and add this to the numerator.

SAMPLE PROBLEM 4 DEMONSTRATES BORROWING WHEN SUBTRACTING MIXED NUMBERS.

SAMPLE PROBLEM 4 WITH SOLUTION

Perform the indicated subtraction, then reduce the resulting fractions or mixed numbers to lowest terms.		
a. $8^{1}/_{8} - 3^{5}/_{6}$	b. $7 - 4^{12}/_{16}$	
$8\frac{1}{8} = 8\frac{1\times3}{8\times3} = 8\frac{3}{24} = 7\frac{27}{24}$ $-3\frac{5}{6} = -3\frac{5\times4}{6\times4} = -3\frac{20}{24} = -3\frac{20}{24}$ $-3\frac{7}{24} = -3\frac{20}{24}$	$7 = 7\frac{0}{16} = 6\frac{16}{16}$ $-4\frac{12}{16} = -4\frac{12}{16} = -4\frac{12}{16}$ $2\frac{4}{16} = 2\frac{1}{4}$	

A. Perform the indicated subtraction, then <u>reduce the resulting fractions or mixed numbers to</u> <u>lowest terms</u>. Refer to Sample Problem 4, as needed. For more practice, see pages 34 – 36 of Contemporary's <u>Number Power 2</u> work-text.

$$2\frac{3}{8} \qquad 5 \qquad 12\frac{2}{7}$$

$$1. \frac{-\frac{5}{8}}{2} \qquad 2. \frac{-1\frac{3}{5}}{5} \qquad 3. \frac{-9\frac{2}{5}}{5}$$

$$4. 6-3^{5}/9= \qquad 5. 8^{1}/8-5^{5}/16= \qquad 6. 1^{11}/20-5^{5}/6=$$

VI. ADDITION & SUBTRACTION PROBLEMS INVOLVING FRACTIONS WITH UNLIKE DENOMINATORS

- A. <u>Completely solve</u> the problems below using steps (*i*), (*ii*), and (*iii*) below. Refer to Sample Problem 5 on the next page, as needed. Additional practice problems can be found on pages 29, 30, 37 and 38 of Contemporary's Number Power 2 work-text.
 - *i*. State what it is you are to find. Give your answer as a complete sentence.
 - *ii*. Solve the problem, showing your work.
 - *iii*. State the answer in a complete sentence.
 - 1. Jill needs to ship three packages weighing $2^{3}/_{4}$ lb., $3^{1}/_{8}$ lb., and $1^{2}/_{3}$ lb. What is the total weight that Jill will be shipping?
 - 2. Jeff is making a bookshelf. To make one shelf he is going to cut a piece $32^{5}/_{16}$ inches long from a board that measures $95^{1}/_{2}$ inches. How long will the remaining piece be?
 - 3. Larry bought a ten-pound bag of rice for some upcoming catering jobs. If he used $2^{5}/_{8}$ lb. of rice on the first job, how much rice does he have left?
 - 4. Mary has a part-time job with flexible hours. Last week she worked $4^{1/3}$ hours on Tuesday, $3^{1/2}$ hours on Thursday, and $4^{3/4}$ hours on Saturday. In all, how many hours did Mary work last week?

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SAMPLE PROBLEM 5 DEMONSTRATES HOW TO SOLVE PROBLEMS WITH UNLIKE DENONINATORS.

SAMPLE PROBLEM 5 WITH SOLUTION

The Problem:

To figure out how much fuel has been added to a race car the pit crew compares the weight of the gas can before the pit stop to it's weight after the stop. How many pounds of fuel were added to the car's tank if the weight of the can was $73^3/_8$ lb. before the fill-up and $18^{3}/_{4}$ lb. afterwards?

The Solution:

- *i*. We are to find how many pounds of fuel were added to the car's tank.
- ii. To find how many pounds of fuel were added to the car's tank we need to subtract the weight of the can after the fueling $(18^{3}/_{4} \text{ lb.})$ from the weight of the can before the fueling $(73^3/_8 \text{ lb.})$. One estimate is 55 lb. (75 lb. – 20 lb.).

The calculation at the right $73^{3}/_{8}$ lb. = $73^{3}/_{8}$ lb. = $72^{11}/_{8}$ lb. gives the exact amount needed $-\frac{18^{3}}{_{4}}$ lb. = $-\frac{18^{6}}{_{8}}$ lb. = $-\frac{18^{6}}{_{54}}$ lb. = $-\frac{18^{6}}{_{54}}$ lb. = $-\frac{18^{6}}{_{54}}$ lb. = $-\frac{18^{6}}{_{54}}$ lb.

iii. The crew added $54^{5}/_{8}$ pounds of gas to the car's tank.

ANSWER KEY

SECTION I: REVIEW PROBLEMS

A1. $\frac{2}{6}$ or $\frac{1}{3}$ A2. $\frac{3}{5}$ A3. $\frac{6}{16}$ or $\frac{3}{8}$ B1. $\frac{3}{20}$ B2. $\frac{11}{51}$ B3. $\frac{25}{66}$ C1. Improper C2. Mixed C3. Proper C4. Proper C5. Improper D1. $2^{1}/_{4}$ D2. 3 D3. $1^{1}/_{3}$ D4. $1^{8}/_{9}$ D5. 4 E1. $4^{4}/_{3}$ E2. $2^{29}/_{5}$ E3. $2^{5}/_{4}$ E4. $4^{1}/_{4}$ E5. $2^{8}/_{11}$ F1. ? = 6 F2. ? = 28 F3. ? = 54 F4. ? = 85 G1. $2^{1}/_{16}$ in. **SECTION II: DIVISIBILITY RULES** A3. no B1. yes B2. yes A1. yes A2. yes B3. no C1. yes C2. no C3. yes SECTION III: ADD & SUBTRACT FRACTIONS & MIXED NUMBERS WITH UNLIKE DENOMINATORS A1. 8 A2. 18 A3. 40 B3. $6^{1}/_{2}$ B4. $9^{1}/_{10}$ B5. $5^{1}/_{8}$ B6. $15^{23}/_{24}$ B2. 5/6B1. $^{21}/_{80}$ SECTION IV: CARRYING WHEN ADDING FRACTIONS & MIXED NUMBERS A1. $1^{11}/_{16}$ A2. $9^{107}/_{120}$ A3. $50^{21}/_{40}$ A4. $7^{4}/_{75}$ A5. $1^{5}/_{12}$ A5. $68^{13}/_{30}$ SECTION V: BORROWING WHEN SUBTRACTING FRACTIONS & MIXED NUMBERS A1 $1^{3}/_{4}$ A2. $3^{2}/_{5}$ A3. $2^{31}/_{35}$ A4. $2^{4}/_{9}$ A5. $2^{13}/_{16}$ A6. $^{43}/_{60}$ SECTION VI: ADDITION & SUBTRACTION PROBLEMS INVOLVING UNLIKE DENOMINATORS A1. $7^{13}/_{24}$ lb. A2. $63^{3}/_{16}$ in. A3. $7^{3}/_{8}$ lb. A4. $12^{7}/_{12}$ Created by R. E. Buzby, Sidney, ME -- 1998-2002 Pg. 6 of 6

FRACTION FRAMEWORK ADDITION EXAMPLE

SAMPLE PROBLEM 1 (Fraction Addition):

$$\frac{7}{-10} + \frac{7}{-15} =$$

<u>STEP 1</u>:

Convert the problem layout from horizontal to vertical.

	7	
	—	
	10	
	7	
F		
	15	

<u>STEP 2</u>: Determine the Least Common Denominator (LCD).

- Check to see if the smaller denominator, 10, divides evenly into the larger denominator, 15. It DOES NOT, so ...
- Check to see if the denominators have any common factors. If they DO NOT, the LCD is found by multiplying the denominators. However, both 10 & 15 divide evenly by 5, so ...
- 3. Check other multiples of 15, in order, until we find one that will divide evenly by 10.

2 x 15 = 30, which DOES divide by 10. (30 \div 10 = 3). Therefore, 30 is our LCD.

<u>STEP 3</u>:

Create a framework similar to that shown below, placing the LCD under the fraction bar of your "target" fraction. *[This is the one all the way to the right.]*



<u>STEP 4</u>:

For each fraction, find the number you have to multiply it's original denominator by to get the LCD your "target" fraction. Write this number both above and below the fraction bar of the MIDDLE *[multiplier]* fraction.

[We can do this because "any number divided by itself is 1" and "multiplying any number by 1 does not change its value."]

For this example:

- The top fraction multiplier is 3, since $10 \times 3 = 30$.
- The bottom fraction multiplier is 2, since $15 \times 2 = 30$.

Our framework at this point is shown below.

7	3	
—	x — =	
10	3	30
7	2	
+ —	х — =	+
15	2	30
		_

(Continued on next page)

Framework Examples for Adding & Subtracting Fractions & Mixed Numbers

<u>STEP 5</u>:

For each fraction, multiply it's original numerator by the numerator of the MIDDLE *[multiplier]* fraction and write the result as the numerator of your "target" fraction.

For this example:

- For the top fraction: $7 \times 3 = 21$.
- For the bottom fraction: $7 \times 2 = 14$.

Our framework at this point is shown below.



<u>STEP 6</u>:

Add the numerators of your "target" fractions and write the resulting fraction below your "target" fractions.

Our framework at this point is shown below.



<u>STEP 7</u>:

If the resulting fraction is not already in lowest terms, reduce it by dividing it's numerator and denominator by their common factor(s).

In this problem, the numerator (35) and denominator (30) have a common factor of 5.



<u>STEP 8</u>:

If the resulting fraction is an improper fraction, change it to a mixed number by dividing the numerator by the denominator. *[This can be done using long division as demonstated in the box to the right of the problem.]*

In the end, our final solution looks somerhing like this:



FRACTION FRAMEWORK SUBTRACTION EXAMPLE

SAMPLE PROBLEM 2 (Fraction Subtraction):

$$\frac{13}{-} - \frac{5}{-} = \frac{16}{8}$$

<u>STEP 1</u>:

Convert the problem layout from horizontal to vertical.

13	
_	
16	
5	
8	

<u>STEP 2:</u>

Determine the Least Common Denominator (LCD).

 Check to see if the smaller denominator, 8, divides evenly into the larger denominator, 16. It DOES; therefore, 16 is our LCD and we do not have to look any further.

<u>STEP 3</u>:

Create a framework similar to that shown below, placing the LCD under the fraction bar of your "target" fraction. *(This is the one all the way to the right.)*



<u>STEP 4</u>:

For each fraction, find the number you have to multiply it's original denominator by to get the LCD your "target" fraction. Write this number both above and below the fraction bar of the MIDDLE *[multiplier]* fraction.

[We can do this because "any number divided by itself is 1" and "multiplying any number by 1 does not change its value."]

For this example:

- The top fraction multiplier is 1, since $16 \times 1 = 16$.

- The bottom fraction multiplier is 2, since $8 \times 2 = 16$.

Our framework at this point is shown below.

13	1	
— х	=	
16	1	16
5	2	
- — x	=	
8	2	16

Framework Examples for Adding & Subtracting Fractions & Mixed Numbers

<u>STEP 5</u>:

For each fraction, multiply it's original numerator by the numerator of the MIDDLE *[multiplier]* fraction and write the result as the numerator of your "target" fraction.

For this example:

- For the top fraction: $13 \times 1 = 13$.
- For the bottom fraction: $5 \times 2 = 10$.

Our framework at this point is shown below.

13	1	13
— х	: =	
16	1	16
5	2	10
– — х	: =	
8	2	16

<u>STEP 6</u>:

Subtract the numerators of your "target" fractions and write the resulting fraction below your "target" fractions.

Our framework at this point is shown below.

13	1	13
— х	— =	
16	1	16
5	2	10
- — x	— =	
8	2	16
		3
		16

<u>STEP 7</u>:

If the resulting fraction is not already in lowest terms, reduce it by dividing it's numerator and denominator by their common factor(s).

In this problem, the numerator (3) and denominator (16) have no factors in common.

[The only number that divides evenly into 3 is 3; and, 3 will not divides evenly into 16.]

Thus, our final solution is:

13	1	13
— х	= =	
16	1	16
5	2	10
- — x	=	
8	2	16
		3
		16